Quiz 6

February 6, 2016

Solve the following differential equation for $y(x)$ using an integrating factor, $u(x)$:

$$(\cos x)\frac{dy}{dx} + (\sin x)y = 2\cos^3 x \sin x, \text{ where } 0 \leq x \leq \pi/2$$

Solution:

$$\frac{dy}{dx} + \frac{\sin x}{\cos x}y = 2\cos^2 x \sin x \qquad \text{(divide by } \cos x\text{)}$$

$$\begin{aligned} u(x) &= e^{\int \frac{\sin x}{\cos x} dx} && (u(x) = e^{\int P(x)dx}) \\ &= e^{-\ln|\cos x|} && \text{(u-substitution, } u = \cos x\text{)} \\ &= e^{-\ln(\cos x)} && (\cos x \geq 0 \text{ when } 0 \leq x \leq \pi/2) \\ &= \frac{1}{\cos x} && \text{(since } \ln x \text{ and } e^x \text{ are inverses)} \end{aligned}$$

$$\begin{aligned} \frac{1}{\cos x}y &= \int \left(\frac{1}{\cos x}\right)(2\cos^2 x \sin x)dx && \left(u(x)y = \int u(x)Q(x)dx\right) \\ &= \int 2\cos x \sin x dx \\ &= \sin^2 x + C && (u = \sin x, du = \cos x dx) \\ OR &= -\cos^2 x + C && (u = \cos x, du = -\sin x dx) \\ y &= \sin^2 x \cos x + C\cos x && \text{(multiply by } \cos x\text{)} \\ OR &= -\cos^3 x + C\cos x && \text{(multiply by } \cos x\text{)} \end{aligned}$$